

Optimal Investment and Premium Policies under Risk Shifting and Solvency Regulation*

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Abstract

Limited liability creates a conflict of interests between policyholders and shareholders of insurance companies. It provides shareholders with incentives to increase the risk of the insurer's assets and liabilities which, in turn, might reduce the value policyholders attach to and premiums they are willing to pay for insurance coverage.

We characterize Pareto optimal investment and premium policies in this context and provide necessary and sufficient conditions for their existence and uniqueness. We then identify investment and premium policies under the risk shifting problem if shareholders cannot credibly commit to an investment strategy before policies are sold and premiums are paid. Last, we analyze the effect of solvency regulation, such as Solvency II or the Swiss Solvency Test, on the agency cost of the risk shifting problem and calibrate our model to a non-life insurer average portfolio.

Keywords: Risk Shifting, Insurance, Regulation, Pareto Optimality

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1 Introduction

Risk shifting is a well-known agency problem in corporate finance between shareholders and bondholders of a corporation (Jensen and Meckling [9], Green [8], MacMinn [10]). The limited liability protection provides incentives for management acting in the interests of shareholders to select riskier projects at the expense of bondholders. If management cannot credibly commit not to undertake those projects, bondholders demand an appropriate interest rate differential which reflects the agency cost.

Policyholders of a stock insurance company face a situation similar to that of bondholders of a corporation. By paying premiums, policyholders provide capital which is senior to equity but under the investment decision of management acting in the interest of shareholders. Limited liability provides incentives for management to increase the risk of the insurer's assets and liabilities by, for example, increasing the risk of the asset portfolio, selling additional policies without a corresponding injection of equity, or by changing the reinsurance arrangements or strategies in asset liability management accordingly. This increase in the stock insurer's risk may raise the insolvency probability of the insurer and consequently reduce the value policyholders attach to and the premium they are willing to pay for the insurance contract.

We provide a formal framework to study the conflict of interest between risk-neutral shareholders of an insurance corporation and its risk-averse policyholders. Shareholders have access to a technology through which they can increase the risk of the insurer's assets. We represent the technology by the possibility of investing the total capital of the insurer, including equity and premium payments, in a risky asset. Different risk profiles of the insurer's assets are identified by different fractions of the insurer's total capital invested in the risky asset. While increasing the risk of the insurer's assets may serve shareholders' interests, it may reduce the premium levels policyholders are willing to pay and thereby the total capital available for shareholders to invest.

In this context, we characterize the set of Pareto optimal investment and premium policies. We show that, for any policyholders' or shareholders' reservation utility level, there exists a Pareto optimum and, under some mild assumption, it is unique. Moreover, we specify the necessary and sufficient condition for the Pareto optimum.

We then investigate the risk shifting problem between shareholders and policyholders, that is, the setting in which shareholders cannot credibly commit to a specific investment strategy

before policies are sold and premiums are paid. This agency problem generically leads to Pareto suboptimal investment policies and corresponding premium levels. The investment technology implies that the risk shifting problem admits only the boundary solution where the entire insurer's capital is invested in the risky asset.

Last, we analyze the effect of solvency regulation in the context of this agency problem. Solvency regulation imposes a constraint on the set of possible investment and premium policies. We model the regulatory constraint by some general convex risk measure and characterize the corresponding solution. We show that there exists a unique investment strategy and premium level that solves the risk shifting problem under the regulatory constraint and analyze its effect on the inefficiency of the risk shifting problem. Finally, we calibrate our model to an European Economic Area non-life insurer average portfolio taken from the QIS3 (Quantitative Impact Study 3) Benchmarking Study [4] of the Chief Risk Officer Forum under the Solvency II standard model [2, 3] and illustrate our analytical results.

Our paper relates to the literature on the risk shifting problem in corporate finance (Jensen and Meckling [9]). Green [8] presents a formal model in which entrepreneurs decide on the allocation of funds across two mutually non-exclusive projects where one project is riskier than the other in the sense of Rothschild and Stiglitz [18]. The investment technology exhibits some scale function which is strictly concave in the amount invested in each project. The author shows that the agency conflict leads to an overinvestment in the riskier project and discusses the role of convertible bonds in eliminating the risk shifting problem. MacMinn [10] analyzes the risk shifting problem with two mutually exclusive projects but with a linear scale function. The author shows that a sufficiently high level of leverage is necessary to induce the firm to switch from the less risky to the riskier project and thereby to generate an agency cost. Furthermore, there is a set of convertible contracts that eliminate the risk shifting problem. We contribute to this literature by analyzing the risk shifting problem in the insurance setting where the two parties have differing costs of bearing risk and which includes insurance losses as an additional source of risk. Moreover, we examine the effect of solvency regulation on the agency cost of the risk shifting problem. The investment technology in our model can be interpreted as two projects which are mutually non-exclusive (as in Green [8]) and exhibit a linear scale function (as in MacMinn [10]).

Our paper also relates to the insurance literature that discusses the conflicts of interest between shareholders and policyholders of a stock insurer. Mayers and Smith [12], [13] discuss an agency

problem in the context of dividend policies. After policies have been sold, shareholders have an incentive to increase the value of their claim by raising dividends at the expense of policyholders. They argue that the mutual organizational form of an insurance company can help internalize this agency problem. Doherty [5] shows how an increase in the risk of the insurer's asset portfolio increases the shareholders' position at the expense of policyholders. We contribute to this literature by providing a formal model to investigate this conflict of interest and examine the effect of solvency regulation under the agency problem. This framework allows us to determine the specific conditions for existence and uniqueness of Pareto optimal risk structures and premium levels, for the solutions under the agency problem, and under solvency regulation.

Last, we contribute to the literature that analyzes the effect of regulation in insurance markets. Munch and Smallwood [16] and Finsinger and Pauly [7] analyze the optimal amount of shareholder capital and investment risk of an insurance company under the assumption that shareholders cannot influence the risk structure of the insurer's assets and that insurance premiums are independent of the insurer's insolvency risk. Munch and Smallwood [16] argue that regulation may reduce the insolvency risk if shareholder capital is low. Finsinger and Pauly [7], however, argue that regulation of insurance companies may be unnecessary in the long run with building up reserves. McCabe and Witt [14] and MacMinn and Witt [11] analyze the effect of different regulatory schemes on the optimal investment and underwriting activity of an insurer under the assumption that the demand function for insurance is independent of the insurer's probability of insolvency. Rees et al. [17] relax this assumption and show that if policyholders are fully informed about the insurer's insolvency risk then regulation serves no purpose. In our framework, shareholders decide on the investment strategy, policyholders are perfectly informed or, under the agency problem, have rational expectations about the corresponding risk, and premiums therefore depend on the implied insolvency risk. Moreover, we examine the effect of solvency regulation on the agency cost.

The paper is structured as follows. We set up our model in Section 2 and characterize the set of Pareto optimal policies in Section 3. In Section 4, we examine the risk shifting problem and solvency regulation. In Section 5, we calibrate our model to data and illustrate our results. We conclude in Section 6.

2 Setup

We consider a one-period economy with two agents, a policyholder and a shareholder. The policyholder is endowed with some initial wealth w_0 and faces a random loss X . His preferences are characterized by some Bernoulli utility function $u : \mathbb{R} \rightarrow \mathbb{R}$.

The shareholder is risk-neutral. He owns a stock insurance company with initial capital $c_0 > 0$ which offers full insurance coverage for X in exchange for a premium p . The shareholder has access to a technology which allows him to increase the insurer's risk. We represent this technology by an investment opportunity in a risky asset that yields a random return R . The shareholder thus decides on the fraction $\alpha \in [0, 1]$ of the total capital, $c_0 + p$, to be invested in the risky asset. The risk-free interest rate is assumed to be zero or, equivalently, all values are in units of the risk-free numeraire. We assume that the shareholder has only access to the investment technology if he sells insurance. He may thus be willing to accept a negative premium $p > -c_0$ to gain access to the investment technology. We denote the set of investment and premium policies (α, p) by $\mathcal{P} = [0, 1] \times (-c_0, \infty)$.

The investment decision of the shareholder can be interpreted as an allocation decision of funds across two projects, a risk-free project with return zero and a risky project with random return R . Moreover, the two projects are mutually non-exclusive (as in Green [8]) and exhibit a linear scale function (as in MacMinn [10]). For $\mathbb{E}[R] = 0$, the project with return R is riskier in the sense of Rothschild and Stiglitz [18]. Allocating a higher fraction α of the total capital, $c_0 + p$, to the risky asset thus increases the risk of the insurer's assets and liabilities. This can be achieved, for example, by increasing the risk of the insurer's asset portfolio, by reducing the duration matching of assets and liabilities, or by increasing the attachment point of reinsurance contracts.

The end of the period surplus is given by $(c_0 + p)(1 + \alpha R) - X$. If the surplus is negative, the insurance company is insolvent and the shareholder is protected by limited liability. In this case, the policyholder receives the remaining assets, $(c_0 + p)(1 + \alpha R)$. Consequently, the terminal payoff to the shareholder equals

$$((c_0 + p)(1 + \alpha R) - X)^+$$

while the terminal wealth of the policyholder is given by

$$w_0 - p - (X - (c_0 + p)(1 + \alpha R))^+.$$

The corresponding utility of the shareholder and the policyholder as a function of α and p is

$$U_{SH}(\alpha, p) = \mathbb{E} \left[((c_0 + p)(1 + \alpha R) - X)^+ \right]$$

and

$$U_{PH}(\alpha, p) = \mathbb{E} \left[u \left(w_0 - p - (X - (c_0 + p)(1 + \alpha R))^+ \right) \right],$$

respectively.

Assumption 2.1. *Throughout the paper, we make the following standing assumptions:*

- (i) *u is increasing, concave, and twice differentiable on \mathbb{R} with $u' > 0$, $u'' < 0$, $\lim_{x \rightarrow -\infty} u(x) = -\infty$ and $\lim_{x \rightarrow -\infty} u'(x) = \infty$.*
- (ii) *(X, R) takes values in $\mathbb{R}_+ \times [-1, \infty)$ and admits a jointly continuous density function $f(x, r)$.*
- (iii) *The solvency event $\mathcal{S}(\alpha, p) = \{(c_0 + p)(1 + \alpha R) \geq X\}$ has positive probability, $\mathbb{P}[\mathcal{S}(\alpha, p)] > 0$, for all $(\alpha, p) \in \mathcal{P}$.*
- (iv) *u and f are such that U_{SH} and U_{PH} are real-valued and differentiable in some neighborhood of \mathcal{P} , and the following formal manipulations (e.g. changing the order of differentiation and integration) are meaningful.¹*

Lemmas A.1–A.3 in the appendix illustrate the qualitative behavior of the shareholder and policyholder utility function on \mathcal{P} . In the sequel we will draw on these results without further mention. From Lemma A.1 we know that $U_{PH}(1, p)$ is strictly concave in p . Hence there exists a unique critical premium level $p_{PH}^{crit} \in [-c_0, \infty)$ which maximizes the policyholder utility for $\alpha = 1$,

$$U_{PH}(1, p_{PH}^{crit}) = \max_{p \in [-c_0, \infty)} U_{PH}(1, p).$$

We denote the corresponding critical shareholder and policyholder utility levels by $\gamma_{SH}^{crit} = U_{SH}(1, p_{PH}^{crit})$

¹For example, it is sufficient (but not necessary) that f has compact support.

and $\gamma_{PH}^{crit} = U_{PH}(1, p_{PH}^{crit})$, and define the intervals²

$$\Gamma_{SH} = \begin{cases} (\gamma_{SH}^{crit}, \infty) = (0, \infty), & \text{if } p_{PH}^{crit} = -c_0, \\ [\gamma_{SH}^{crit}, \infty), & \text{if } p_{PH}^{crit} > -c_0, \end{cases}$$

$$\Gamma_{PH} = \begin{cases} (-\infty, \gamma_{PH}^{crit}), & \text{if } p_{PH}^{crit} = -c_0, \\ (-\infty, \gamma_{PH}^{crit}], & \text{if } p_{PH}^{crit} > -c_0. \end{cases}$$

3 Pareto Optimal Investment and Premium Policies

In this setup, we now examine optimal policies $(\alpha, p) \in \mathcal{P}$ in the following sense:

Definition 3.1. *The policy $(\alpha^*, p^*) \in \mathcal{P}$ is Pareto optimal if there does not exist any other policy $(\alpha, p) \in \mathcal{P}$ such that $U_{SH}(\alpha, p) \geq U_{SH}(\alpha^*, p^*)$ and $U_{PH}(\alpha, p) \geq U_{PH}(\alpha^*, p^*)$ with strict inequality for at least one of them.*

We first show that Pareto optimality is equivalent to a constrained optimization problem.

Theorem 3.2. *For any policy $(\alpha^*, p^*) \in \mathcal{P}$, the following are equivalent:*

- (i) (α^*, p^*) is Pareto optimal.
- (ii) (α^*, p^*) solves the constrained optimization problem

$$\begin{aligned} \max_{(\alpha, p) \in \mathcal{P}} U_{PH}(\alpha, p) \\ \text{s.t. } U_{SH}(\alpha, p) \geq \gamma_{SH} \end{aligned} \tag{1}$$

for the shareholder's reservation utility level $\gamma_{SH} = U_{SH}(\alpha^*, p^*)$, and $\gamma_{SH} \in \Gamma_{SH}$.

- (iii) (α^*, p^*) solves the constrained optimization problem

$$\begin{aligned} \max_{(\alpha, p) \in \mathcal{P}} U_{SH}(\alpha, p) \\ \text{s.t. } U_{PH}(\alpha, p) \geq \gamma_{PH} \end{aligned} \tag{2}$$

for the policyholder's reservation utility level $\gamma_{PH} = U_{PH}(\alpha^*, p^*)$, and $\gamma_{PH} \in \Gamma_{PH}$.

²Note that $p_{PH}^{crit} = -c_0$ if and only if $\gamma_{SH}^{crit} = 0$.

Moreover, in either of the above optimization problems, (1) and (2), the respective reservation utility constraint is binding.

Proof. (i) \Rightarrow (ii): let $(\alpha^*, p^*) \in \mathcal{P}$ be Pareto optimal. Then clearly (α^*, p^*) solves the constrained optimization problem (1). It remains to be shown that $\gamma_{SH} \in \Gamma_{SH}$. If $(\alpha^*, p^*) = (1, p_{PH}^{crit})$ or if $p_{PH}^{crit} = -c_0$ then there is nothing to prove. So assume that $(\alpha^*, p^*) \neq (1, p_{PH}^{crit}) \in \mathcal{P}$. In view of Lemma A.3 (iv), we have $U_{PH}(\alpha^*, p^*) < U_{PH}(1, p_{PH}^{crit})$. But then, by Pareto optimality of (α^*, p^*) , we must have $\gamma_{SH} = U_{SH}(\alpha^*, p^*) > U_{SH}(1, p_{PH}^{crit}) = \gamma_{SH}^{crit}$. This proves the claim.

(ii) \Rightarrow (i): let $(\alpha^*, p^*) \in \mathcal{P}$ be a maximizer of (1). We argue by contradiction and assume that (α^*, p^*) is not Pareto optimal. Then there exists some policy $(\bar{\alpha}, \bar{p}) \in \mathcal{P}$ such that $U_{PH}(\bar{\alpha}, \bar{p}) \geq U_{PH}(\alpha^*, p^*)$ and $U_{SH}(\bar{\alpha}, \bar{p}) \geq \gamma_{SH} = U_{SH}(\alpha^*, p^*)$ with strict inequality for at least one of them. If $U_{PH}(\bar{\alpha}, \bar{p}) > U_{PH}(\alpha^*, p^*)$ then clearly (α^*, p^*) cannot be an optimizer of (1). Hence we can assume that $U_{SH}(\bar{\alpha}, \bar{p}) > \gamma_{SH}$ and $U_{PH}(\bar{\alpha}, \bar{p}) = U_{PH}(\alpha^*, p^*)$. Then there exists a neighborhood O of $(\bar{\alpha}, \bar{p})$ in \mathcal{P} such that $U_{SH}(\alpha, p) \geq \gamma_{SH}$ for all $(\alpha, p) \in O$. By Lemma A.2 below, $\nabla U_{PH}(\bar{\alpha}, \bar{p}) \neq 0$, and $\partial_p U_{PH}(\bar{\alpha}, \bar{p}) < 0$ if $\bar{\alpha} = 0$ in particular. Moreover, if $\bar{\alpha} = 1$ then $U_{SH}(1, \bar{p}) > \gamma_{SH} \geq \gamma_{SH}^{crit} = U_{SH}(1, p_{PH}^{crit})$ implies $\bar{p} > p_{PH}^{crit}$ and thus $\partial_p U_{PH}(\bar{\alpha}, \bar{p}) < 0$ again. Hence in either case we conclude that there exists some $(\alpha, p) \in O$ with $U_{PH}(\alpha, p) > U_{PH}(\bar{\alpha}, \bar{p}) \geq U_{PH}(\alpha^*, p^*)$, whence (α^*, p^*) does not solve (1), which is absurd. Hence (α^*, p^*) is Pareto optimal. Moreover, this shows that the reservation utility constraint $U_{SH}(\alpha, p) \geq \gamma_{SH}$ is binding.

The equivalence (i) \Leftrightarrow (iii) follows similarly but simpler, since $\partial_p U_{SH}(\alpha, p) > 0$ by Lemma A.2. \square

Pareto optimal investment and premium policies can thus be generated by a take-it-or-leave-it offer either of the policyholder to the shareholder (optimization problem (1)) or of the shareholder to the policyholder (optimization problem (2)). The participation constraint is binding in either case because both the policyholder's and shareholder's preferences are locally non-satiated.

Note that part (ii) of Theorem 3.2 implies that there exists no Pareto optimal policy for a shareholder utility level $\gamma_{SH} \notin \Gamma_{SH}$. In fact, as for the existence of Pareto optimal policies, we have the following theorem.

Theorem 3.3. *For any reservation utility level $\gamma_{SH} \in \Gamma_{SH}$ and $\gamma_{PH} \in \Gamma_{PH}$, respectively, there exists at least one Pareto optimum $(\alpha^*, p^*) \in \mathcal{P}$ with $U_{SH}(\alpha^*, p^*) = \gamma_{SH}$ and $U_{PH}(\alpha^*, p^*) = \gamma_{PH}$,*

respectively. It satisfies the first order condition

$$\mathbb{E} \left[R u' \left(w_0 - p^* - (X - (c_0 + p^*)(1 + \alpha^* R))^+ \right) \right] \begin{cases} \leq 0, & \text{if } \alpha^* = 0, \\ = 0, & \text{if } 0 < \alpha^* < 1, \\ \geq 0, & \text{if } \alpha^* = 1. \end{cases} \quad (3)$$

Moreover, for any $\alpha^* \in [0, 1]$ there exists at most one Pareto optimum.

Proof. In view of Theorem 3.2, it is enough to consider the optimization problems (1) and (2), respectively. First note that, in view of Lemmas A.1 and A.2, $U_{PH}(\alpha, p)$ is strictly concave and $U_{SH}(\alpha, p)$ is strictly increasing in p , for all $\alpha \in [0, 1]$. Hence, for any fixed $\alpha \in [0, 1]$, there can be at most one Pareto optimum. Moreover, by Lemma A.3, we have $\Gamma_{SH} \subseteq U_{SH}(\mathcal{P})$ and $\Gamma_{PH} = U_{PH}(\mathcal{P})$. Hence the constraint sets in (1) and (2) are non-empty. Lemma A.3 (iii) implies that for any $\gamma_{PH} \in \Gamma_{PH}$ the level set $\{U_{PH} \geq \gamma_{PH}\} \subset \mathcal{P}$ is compact in \mathcal{P} . Since U_{PH} and U_{SH} are continuous on \mathcal{P} , we conclude that the maximum in both optimization problems (1) and (2), and thus the Pareto optimum at the respective reservation utility level, is attained in \mathcal{P} .

For the derivation of the first order condition, it is convenient to introduce the following diffeomorphism:

$$\begin{aligned} \mathcal{P} &\rightarrow \mathcal{V} = \{(v, w) \mid 0 < v \leq w < \infty\} \\ (\alpha, p) &\mapsto (v, w) = ((c_0 + p)\alpha, c_0 + p). \end{aligned}$$

Note that w is the total asset value of the insurer and v is the money invested in the stock market. The corresponding utility of the shareholder and the policyholder as a function of the new coordinates (v, w) is

$$\begin{aligned} V_{SH}(v, w) &= \mathbb{E} \left[(w + vR - X)^+ \right] \\ V_{PH}(v, w) &= \mathbb{E} \left[u \left(w_0 + c_0 - w - (X - w - vR)^+ \right) \right], \end{aligned} \quad (4)$$

so that $V_{SH}(v, w) = U_{SH}(\alpha, p)$ and $V_{PH}(v, w) = U_{PH}(\alpha, p)$. For simplicity of notation, we use the same letter $\mathcal{S}(v, w) = \mathcal{S}(\alpha, p)$ for the respective solvency event.

We note that V_{SH} is a convex and V_{PH} is a concave function jointly in (v, w) . In contrast, U_{SH} and U_{PH} do not share these properties as functions jointly in (α, p) in general.

By Lemma A.4, we have $\partial_w V_{PH} < 0$. Hence, for any $\gamma_{SH} \in \Gamma_{SH}$, the implicit function theorem yields a continuously differentiable function $W : I \rightarrow (0, \infty)$ on some interval $I \subset \mathbb{R}_+$ with $(v, W(v)) \in \mathcal{V}$, $V_{SH}(v, W(v)) = \gamma_{SH}$, and

$$W'(v) = -\frac{\partial_v V_{SH}(v, W(v))}{\partial_w V_{SH}(v, W(v))} = -\frac{\mathbb{E}[R 1_{\mathcal{S}(v, w)}]}{\mathbb{P}[\mathcal{S}(v, w)]}. \quad (5)$$

Since for every fixed $\alpha \in [0, 1]$ the function $V_{SH}(\alpha w, w) = U_{SH}(\alpha, w - c_0)$ is strictly increasing in w and maps the interval $(0, \infty)$ onto itself, we can assume that $I = [0, v']$ for some $v' > 0$, and that $W(0) = 0$ and $W(v') = v'$.

We now characterize the critical points for the policyholder utility function along the level curve $(v, W(v))$. A calculation shows

$$\begin{aligned} \frac{d}{dv} V_{PH}(v, W(v)) &= \partial_v V_{PH}(v, W(v)) + W'(v) \partial_w V_{PH}(v, W(v)) \\ &= \mathbb{E}[R u'(w_0 + c_0 - X + vR) 1_{\mathcal{S}(v, w)^c}] \\ &\quad + \frac{\mathbb{E}[R 1_{\mathcal{S}(v, w)}]}{\mathbb{P}[\mathcal{S}(v, w)]} u'(w_0 + c_0 - w) \mathbb{P}[\mathcal{S}(v, w)] \\ &= \mathbb{E}[R u'(w_0 + c_0 - W(v) - (X - W(v) - vR)^+)]. \end{aligned} \quad (6)$$

Hence any Pareto optimal $(v^*, w^*) \in \mathcal{V}$ satisfies

$$\mathbb{E}[R u'(w_0 + c_0 - w^* - (X - w^* - v^*R)^+)] \begin{cases} \leq 0, & \text{if } v^* = 0, \\ = 0 & \text{if } 0 < v^* < w^*, \\ \geq 0, & \text{if } v^* = w^*. \end{cases}$$

This proves (3). \square

In this theorem, we have shown the existence of Pareto optimal investment and premium policies for any admissible reservation utility level of the policyholder or the shareholder. This result is thus valid for different degrees of competition in the insurance market which can be represented by different reservation utility levels. A higher degree of competition in the insurance market is reflected by a lower reservation utility level of the shareholder. In a perfectly competitive market, the shareholder reservation utility level is given by $\gamma_{SH} = c_0$, derived from his outside option of not selling insurance.

In the following theorem, we specify the condition under which the Pareto optimum is unique and under which the first order condition (3) is also sufficient.

Theorem 3.4. *Assume that $\mathbb{E}[R] \neq 0$ or that, for any $(\alpha, p) \in \mathcal{P}$, either the insolvency event has positive probability, $\mathbb{P}[\mathcal{S}(\alpha, p)^c] > 0$, or $\mathbb{E}[R 1_{\mathcal{S}(\alpha, p)}] \neq 0$. Then for any reservation utility level $\gamma_{SH} \in \Gamma_{SH}$ and $\gamma_{PH} \in \Gamma_{PH}$, respectively, there exists a unique Pareto optimum $(\alpha^*, p^*) \in \mathcal{P}$. Moreover, the first order condition (3) is also sufficient for Pareto optimality of $(\alpha^*, p^*) \in \mathcal{P}$.*

Proof. We use the (v, w) -coordinates as introduced in the proof of Theorem 3.3. Fix $\gamma_{SH} \in \Gamma_{SH}$, and let $W : I \rightarrow (0, \infty)$ be the corresponding level curve as in the proof of Theorem 3.3. In view of (5) and (6), the second derivative of $V_{PH}(v, W(v))$ equals³

$$\begin{aligned} \frac{d^2}{dv^2} V_{PH}(v, W(v)) &= -W'(v) \mathbb{E}[R 1_{\mathcal{S}(v, W(v))}] u''(w_0 + c_0 - W(v)) \\ &\quad + \mathbb{E}[R^2 u''(w_0 + c_0 - X + vR) 1_{\mathcal{S}(v, W(v))^c}] \\ &= \frac{\mathbb{E}[R 1_{\mathcal{S}(v, W(v))}]^2}{\mathbb{P}[\mathcal{S}(v, W(v))]} u''(w_0 + c_0 - W(v)) \\ &\quad + \mathbb{E}[R^2 u''(w_0 + c_0 - X + vR) 1_{\mathcal{S}(v, W(v))^c}], \end{aligned}$$

which under the assumption of the theorem is negative. Indeed, both summands on the right hand side are non-positive. If the insolvency event has positive probability, $\mathbb{P}[\mathcal{S}(v, W(v))^c] > 0$, then the second term is negative. Otherwise $\mathbb{P}[\mathcal{S}(v, W(v))] = 1$ and thus $\mathbb{E}[R 1_{\mathcal{S}(v, W(v))}]^2 = \mathbb{E}[R]^2 > 0$, so that the first term is negative. Hence $V_{SH}(v, W(v))$ is strictly concave in $v \in I$. This proves the theorem. \square

In the proof of this theorem, it is shown that, under very mild assumptions, the policyholder's utility as a function of α along a shareholder's level curve, $U_{SH}(\alpha, p) = \gamma_{SH}$, can only assume three shapes. Either it is strictly decreasing in which case $\alpha^* = 0$ is the Pareto optimal investment policy. Or it is strictly increasing in which case $\alpha^* = 1$ is the Pareto optimal investment policy. Last, it can take a unique inner maximum $0 < \alpha^* < 1$ and is strictly increasing to the left and strictly decreasing to the right. In each of these three cases, the Pareto optimal premium policy p^* is uniquely defined by $U_{SH}(\alpha^*, p^*) = \gamma_{SH}$.

³The differentiation under the expectation sign is justified by Assumption 2.1, see the proof of Lemma A.2 below.

Remark 3.5. Along similar arguments as in the proof of Theorem 3.4, but under the more stringent assumption that $\mathbb{E}[R 1_{\mathcal{S}(\alpha, p)}] > 0$ for all $(\alpha, p) \in \mathcal{P}$, one can show that also the shareholder's utility along a policyholder's level curve, $U_{PH} = \gamma_{PH}$, can only assume the afore mentioned three shapes: increasing, decreasing, or first increasing and then decreasing.⁴ Again, the respective Pareto optimal policy is uniquely defined.

We terminate this section with a generic example where there exists no inner Pareto optimum $(\alpha^*, p^*) \in \mathcal{P}$ with investment policy $\alpha^* > 0$.

Example 3.6. For this example we assume that

- (i) X and R are independent,
- (ii) $\mathbb{E}[R] = 0$,
- (iii) $\mathbb{P}[\mathcal{S}(\alpha, p)^c] > 0$ for all $(\alpha, p) \in \mathcal{P}$.

In particular, the assumption in Theorem 3.4 is satisfied. Now fix an arbitrary policy $(\alpha, p) \in \mathcal{P}$ with $\alpha > 0$. We claim that

$$\mathbb{E}[R u'(w_0 - p - (X - (c_0 + p)(1 + \alpha R))^+)] < 0. \quad (7)$$

This together with the first order condition (3) and Theorem 3.4 then implies that the set of all Pareto optimal policies in \mathcal{P} is given by $\{(0, p^*) \mid p^* \in (-c_0, \infty)\}$. For the proof of (7) we first observe that

$$u'(w_0 - p - (X - (c_0 + p)(1 + \alpha R))^+) \begin{cases} \leq u'(w_0 - p - (X - (c_0 + p))^+) & \text{on } \{R \geq 0\} \\ \geq u'(w_0 - p - (X - (c_0 + p))^+) & \text{on } \{R < 0\} \end{cases}$$

⁴Since $\partial_w V_{PH} < 0$, the implicit function theorem yields a C^1 -policyholder utility level curve $v \mapsto W(v)$ in \mathcal{V} . Some calculations show that $\frac{d}{dv} V_{SH}(v, W(v)) = \frac{\mathbb{E}[R u'(w_0 + c_0 - W(v) - (X - W(v) - vR)^+)]}{u'(w_0 + c_0 - W(v))}$. Further one can show that for critical points v^* where $\frac{d}{dv} V_{SH}(v^*, W(v^*)) = 0$ we have $W'(v^*) < 0$ and

$$\begin{aligned} \frac{d^2}{dv^2} V_{SH}(v, W(v))|_{v=v^*} &= -W'(v^*) \frac{u''(w_0 + c_0 - W(v^*)) \mathbb{E}[R 1_{\mathcal{S}(v^*, W(v^*))}]}{u'(w_0 + c_0 - W(v^*))} \\ &\quad + \frac{\mathbb{E}[R^2 u''(w_0 + c_0 - X + v^* R) 1_{\mathcal{S}(v^*, W(v^*))^c}]}{u'(w_0 + c_0 - W(v^*))}. \end{aligned}$$

From this we conclude that $\frac{d^2}{dv^2} V_{SH}(v, W(v))|_{v=v^*} < 0$. Hence every critical point is a local maximum. This shows that $V_{SH}(v, W(v))$ has the desired properties.

with strict inequalities on $\mathcal{S}(\alpha, p)^c \cap \{R > 0\}$ and $\mathcal{S}(\alpha, p)^c \cap \{R < 0\}$, respectively. Hence

$$R u' (w_0 - p - (X - (c_0 + p)(1 + \alpha R))^+) \leq R u' (w_0 - p - (X - (c_0 + p))^+)$$

with strict inequality on $\mathcal{S}(\alpha, p)^c \cap \{R \neq 0\}$. Since $\mathbb{P}[\mathcal{S}(\alpha, p)^c \cap \{R \neq 0\}] = \mathbb{P}[\mathcal{S}(\alpha, p)^c] > 0$, by the above assumptions, we obtain

$$\begin{aligned} \mathbb{E} [R u' (w_0 - p - (X - (c_0 + p)(1 + \alpha R))^+)] &< \mathbb{E} [R u' (w_0 - p - (X - (c_0 + p))^+)] \\ &= \mathbb{E} [R] \mathbb{E} [u' (w_0 - p - (X - (c_0 + p))^+)] \\ &= 0, \end{aligned}$$

which proves (7).

4 Risk Shifting and Solvency Regulation

In this section, we focus in our analysis of the risk shifting problem on a competitive insurance market. The shareholder is therefore held at his reservation utility level which is derived from his outside option of not selling insurance. We assume that the shareholder has only access to the investment technology if he sells insurance. The shareholder's participation constraint is thus given by $U_{SH}(\alpha, p) \geq c_0$.⁵

Assumption 4.1. *Throughout this section, we make the following standing assumptions:*

- (i) $c_0 \geq \gamma_{SH}^{crit}$,
- (ii) $\mathbb{E}[R \mid X \leq x] > 0$ for all $x \in (0, \infty)$.

Lemma 4.2. *Assumption 4.1(ii) is equivalent to $\mathbb{E}[R 1_{\mathcal{S}(\alpha, p)}] > 0$ for all $(\alpha, p) \in \mathcal{P}$, which again is equivalent to*

$$\partial_\alpha U_{SH}(\alpha, p) > 0 \quad \text{for all } (\alpha, p) \in \mathcal{P}. \tag{8}$$

In this case, the assumption in Theorem 3.4 is satisfied.

⁵Although we focus on a competitive insurance market, we show that all results also hold under the dual problem with the policyholder's participation constraint. The results are thus valid for different degrees of competition.

Proof. It follows by inspection that $\mathbb{E}[R 1_{\{X \leq c_0 + p\}}] \leq \mathbb{E}[R 1_{\mathcal{S}(\alpha, p)}]$ for all $(\alpha, p) \in \mathcal{P}$. In view of Lemma A.2, Assumption 4.1(ii) implies (8). Conversely, that (8) implies (ii) follows from setting $\alpha = 0$ in (8). Moreover, it follows that the assumption in Theorem 3.4 is satisfied. \square

Remark 4.3. *Assumption 4.1(ii) states that the conditional expected return of the risky asset given bounded insurance losses is positive. From a risk management point of view, it is important to note that this moderate assumption is compatible with stress scenarios of negative expected returns under catastrophic insurance losses. A sufficient condition for (ii) is*

$$\mathbb{E}[R \mid X = x] > 0 \quad \text{for all } \text{ess inf } X < x < \text{ess sup } X.$$

In particular, this sufficient condition holds if the risky asset has a positive unconditional expected return, $\mathbb{E}[R] > 0$, and R and X are independent.

We now consider the following sequence of events. The policyholder pays a premium p in exchange for full insurance coverage of X . The shareholder then decides on the fraction α of the total capital, $c_0 + p$, to be invested in the risky asset.

If the shareholder can commit to an investment policy α before the insurance premium p is paid, then the optimal investment and premium policy is given by the solution to the following optimization problem:

$$\begin{aligned} \max_{(\alpha, p) \in \mathcal{P}} & U_{PH}(\alpha, p) \\ \text{s.t. } & U_{SH}(\alpha, p) \geq \gamma_{SH} \end{aligned} \tag{9}$$

for the shareholder's reservation utility level $\gamma_{SH} = c_0$. By Theorem 3.2 and Assumption 4.1, the solution to (9) is Pareto optimal.

We now examine the situation in which the shareholder cannot credibly commit to an investment policy α before the insurance premium p is paid. That is, after the policyholder has paid the insurance premium p , the shareholder chooses α by solving the following optimization problem:

$$\max_{\alpha \in [0, 1]} U_{SH}(\alpha, p). \tag{10}$$

By Lemma 4.2, for any fixed $p \in (-c_0, \infty)$, $U_{SH}(\alpha, p)$ is strictly increasing in α . Hence only the boundary solution is possible, i.e. $\arg \max_{\alpha} U_{SH}(\alpha, p) = 1$. The shareholder invests the entire total

capital in the risky asset.⁶

The policyholder has rational expectations about this investment strategy and the premium p has to satisfy the incentive compatibility constraint that the shareholder sets the investment strategy according to optimization problem (10). We can thus formalize the agency problem by the following constrained optimization problem:

$$\begin{aligned} \max_{(\alpha, p) \in \mathcal{P}} \quad & U_{PH}(\alpha, p) \\ \text{s.t.} \quad & U_{SH}(\alpha, p) \geq \gamma_{SH}, \\ & \alpha \in \arg \max_{\alpha'} U_{SH}(\alpha', p) \end{aligned} \tag{11}$$

for the shareholder's reservation utility level $\gamma_{SH} = c_0$. In view of Theorem 3.2 (ii), any solution of the risk shifting problem (11) is generically Pareto suboptimal.

In the following theorem, we characterize the solution and show that it can be equivalently derived from maximal shareholder utility constraining on the policyholder's reservation utility level.⁷

Theorem 4.4. *For any reservation utility level $\gamma_{SH} \in \Gamma_{SH}$ and $\gamma_{PH} \in \Gamma_{PH}$, respectively, there exists a unique solution $(\bar{\alpha}, \bar{p})$ in \mathcal{P} to the constrained optimization problem (11) and to the dual problem*

$$\begin{aligned} \max_{(\alpha, p) \in \mathcal{P}} \quad & U_{SH}(\alpha, p) \\ \text{s.t.} \quad & U_{PH}(\alpha, p) \geq \gamma_{PH}, \\ & \alpha \in \arg \max_{\alpha'} U_{SH}(\alpha', p) \end{aligned} \tag{12}$$

respectively. This solution satisfies $\bar{\alpha} = 1$, and $U_{SH}(1, \bar{p}) = \gamma_{SH}$ and $U_{PH}(1, \bar{p}) = \gamma_{PH}$, respectively. It is Pareto optimal if and only if $\mathbb{E}[R u'(w_0 - \bar{p} - (X - (c_0 + \bar{p}))(1 + R))^+] \geq 0$.

Moreover, any solution $(\bar{\alpha}, \bar{p})$ to (11) with $\gamma_{SH} = U_{SH}(\bar{\alpha}, \bar{p}) \in \Gamma_{SH}$ is a solution to (12) with $\gamma_{PH} = U_{PH}(\bar{\alpha}, \bar{p}) \in \Gamma_{PH}$, and vice versa.

Proof. As seen below (10), for any fixed $p \in (-c_0, \infty)$, we have $\arg \max_{\alpha'} U_{SH}(\alpha', p) = 1$. Hence

⁶There is only the boundary solution under the risk shifting problem in our setting due to two features of the investment technology. First, any fraction $\alpha \in [0, 1]$ can be invested in the risky asset. Second, the investment technology exhibits a linear scale function, i.e., there are no costs other than the agency cost involved in shifting risk. A strictly concave scale function, as assumed in Green [8] with mutually non-exclusive projects, would imply that the shareholder overinvests in risk but not necessarily up to the boundary.

⁷The results thus also apply to other degrees of competition, including a monopolistic insurance market which is given by the policyholder's reservation utility level derived from his outside option of not buying insurance, i.e. $\gamma_{PH} = \mathbb{E}[u(w_0 - X)]$. Note that this is covered by the theorem since $\mathbb{E}[u(w_0 - X)] < \mathbb{E}[u(w_0 + c_0 - X)] \leq \gamma_{PH}^{crit}$.

problem (11) boils down to optimizing the continuous function $U_{PH}(1, p)$ on the interval $[\bar{p}, \infty)$ with \bar{p} determined by $U_{SH}(1, \bar{p}) = \gamma_{SH}$. Note that $\gamma_{SH} \in \Gamma_{SH}$ is equivalent to $\bar{p} \geq p_{PH}^{crit}$. Thus, $U_{PH}(1, p)$ is strictly decreasing in $p \in [\bar{p}, \infty)$. The optimizer of (11) is thus $(1, \bar{p})$. The first order condition for Pareto optimality follows from Theorem 3.4, see Lemma 4.2. A similar argument applies to problem (12). We conclude, in particular, that any solution $(1, \bar{p})$ to either (11) with $\gamma_{SH} \in \Gamma_{SH}$ or (12) with $\gamma_{PH} \in \Gamma_{PH}$ satisfies $\bar{p} \geq p_{PH}^{crit}$. The last statement thus follows by the fact that $U_{PH}(1, p)$ and $U_{SH}(1, p)$ are strictly decreasing and increasing in $p \in [p_{PH}^{crit}, \infty)$, respectively. \square

The inefficiency of the investment and premium policy arises if shareholders cannot credibly commit to an investment strategy before premiums are paid. Any credible commitment device for shareholders would thus increase welfare. In the context of corporate finance, Green [8] and MacMinn [10] have shown that issuing convertible bonds can provide such a commitment device and eliminate the risk shifting problem.

In our context of insurance there are also various contractual and organizational features that might provide some form of commitment device. Shareholders can limit the insurer's risk exposure by including restrictions on their investment and dividend policies in their corporate charter. Shareholders could also transfer parts of their assets to an escrow account. These restrictions limit the extent to which shareholders can increase the insurer's risk structure and thereby reduce the inefficiency caused by the risk shifting problem. Along the lines of convertible bonds, participating policies reduce the benefit to shareholders from increasing the insurer's risk. The implied benefit of partially overcoming the risk shifting problem might outweigh the cost of higher exposure for policyholders to the insurer's risk. Last, changing the organizational form to a mutual form would eliminate the risk shifting problem. Under the mutual form, owners who decide on the insurer's risk structure coincide with providers of capital. The incentive problem is thereby eliminated at the cost of less diversified owners.

In the following, we take the insurance contract as given and explore the effect of solvency regulation on the agency cost of the risk shifting problem. The regulator assesses the riskiness of the annual loss

$$L(\alpha, p) = c_0 - ((c_0 + p)(1 + \alpha R) - X) = -(c_0 + p)\alpha R + X - p$$

by means of a risk measure ρ . The respective regulatory requirement is that the available capital, c_0 , be greater than the required capital, $\rho(L(\alpha, p))$:

$$\rho(L(\alpha, p)) \leq c_0.$$

This capital requirement restricts the set of feasible investment and premium policies. Solvency capital requirement can thus be interpreted as a commitment device for shareholders imposed by the regulator.

The risk shifting problem (11) under this additional regulatory constraint is as follows

$$\begin{aligned} & \max_{(\alpha, p) \in \mathcal{P}} U_{PH}(\alpha, p) \\ & \text{s.t. } U_{SH}(\alpha, p) \geq \gamma_{SH}, \\ & \alpha \in \arg \max_{\alpha'} U_{SH}(\alpha', p) \\ & \text{s.t. } \rho(L(\alpha', p)) \leq c_0 \end{aligned} \tag{13}$$

for the shareholder's reservation utility level $\gamma_{SH} = c_0$.

We make the following standard assumptions for risk measures, see e.g. McNeil et al. [15].

Assumption 4.5. *Throughout, ρ satisfies the following conditions:*

(i) ρ is cash-invariant, that is, $\rho(L + c) = \rho(L) + c$ for any constant cash amount $c \in \mathbb{R}$ and random loss L .

(ii) ρ is convex, that is,

$$\rho(\lambda L + (1 - \lambda)L') \leq \lambda \rho(L) + (1 - \lambda)\rho(L')$$

for all $\lambda \in [0, 1]$ and random losses L, L' .

(iii) ρ is monotone, that is, $\rho(L) \leq \rho(L')$ if $L \leq L'$.

(iv) $\rho(L(\alpha, p))$ is continuous in $\alpha \in [0, 1]$ for all $p \in (-c_0, \infty)$.

The cash-invariance property of ρ is motivated by its interpretation as regulatory capital requirement. Adding a deterministic cash amount c to the position, the capital requirement is reduced by the same amount. The economic idea behind the convexity assumption of ρ is that diversification by means of combining risks reduces overall risk and therefore the capital requirement.

This assumption is crucial for the constrained problem (13) to be well-posed. See also Remark 4.7 below. The assumption of monotonicity is economically meaningful since an annual loss greater in any state of the world should lead to a higher capital requirement. Property (iv) is of technical nature. It implies that

$$\alpha_\rho(p) = \sup \{ \alpha \in [0, 1] \mid \rho(L(\alpha, p)) \leq c_0 \}$$

satisfies either $\rho(L(\alpha_\rho(p), p)) = c_0$, or $\alpha_\rho(p) = 1$ if $\rho(L(\alpha, p)) \leq c_0$ for all $\alpha \in [0, 1]$, or $\alpha_\rho(p) = -\infty$ if $\rho(L(\alpha, p)) > c_0$ for all $\alpha \in [0, 1]$.⁸ Since the shareholder prefers great α , that is, $\partial_\alpha U_{SH} > 0$, we have that $\alpha_\rho(p)$ equals the arg max in the regulatory constrained subproblem in (13) given that it is well-posed, that is, $\alpha_\rho(p) > -\infty$. Property (iii) implies that $\rho(L(\alpha, p)) \leq \rho(L(\alpha, p'))$ if $p \geq p'$. Hence the domain $\mathcal{D}_\alpha = \{p \in (-c_0, \infty) \mid \alpha_\rho(p) > -\infty\}$ of α_ρ is either empty or an interval with $\sup \mathcal{D}_\alpha = \infty$. We define the corresponding intervals of feasible utility levels

$$\begin{aligned} \Gamma_{SH}^{reg} &= \{U_{SH}(\alpha_\rho(p), p) \mid p \in \mathcal{D}_\alpha\} \\ \Gamma_{PH}^{reg} &= \{U_{PH}(\alpha_\rho(p), p) \mid p \in \mathcal{D}_\alpha\}. \end{aligned}$$

Here is our existence and uniqueness result for the regulatory constrained risk shifting problem (13).

Theorem 4.6. *Assume $\mathcal{D}_\alpha \neq \emptyset$. For any reservation utility level $\gamma_{SH} \in \Gamma_{SH}^{reg}$ and $\gamma_{PH} \in \Gamma_{PH}^{reg}$, respectively, there exists a unique solution $(\hat{\alpha}, \hat{p})$ in \mathcal{P} to the constrained optimization problem (13) and to the dual problem*

$$\begin{aligned} \max_{(\alpha, p) \in \mathcal{P}} \quad & U_{SH}(\alpha, p) \\ \text{s.t.} \quad & U_{PH}(\alpha, p) \geq \gamma_{PH}, \\ & \alpha \in \arg \max_{\alpha'} U_{SH}(\alpha', p) \\ & \text{s.t. } \rho(L(\alpha', p)) \leq c_0 \end{aligned} \tag{14}$$

respectively. This solution satisfies $U_{SH}(\hat{\alpha}, \hat{p}) \geq \gamma_{SH}$ and $U_{PH}(\hat{\alpha}, \hat{p}) = \gamma_{PH}$, respectively, and $\rho(L(\hat{\alpha}, \hat{p})) = c_0$ if $\hat{\alpha} < 1$.

Moreover, any solution $(\hat{\alpha}, \hat{p})$ to (13) with $\gamma_{SH} = U_{SH}(\hat{\alpha}, \hat{p}) \in \Gamma_{SH}^{reg}$ is a solution to (14) with $\gamma_{PH} = U_{PH}(\hat{\alpha}, \hat{p}) \in \Gamma_{PH}^{reg}$, and vice versa.

Proof. We argue in the (v, w) -coordinates introduced in the proof of Theorem 3.3, and define the

⁸Following the usual convention, we define $\sup \emptyset = -\infty$.

corresponding regulator's risk measurement function

$$V_R(v, w) = \rho(-w - vR + X) = -w + \rho(-vR + X)$$

for $0 < v \leq w < \infty$. It follows by inspection that $V_R(v, w) = \rho(L(\alpha, p)) - c_0$, and thus $V_R(v, w)$ is continuous in v . Moreover, $\alpha_\rho(p)$ corresponds to

$$v_\rho(w) = \sup \{v \in [0, w] \mid V_R(v, w) \leq 0\}.$$

In view of Lemmas 4.2 and A.4, we know that $\partial_v V_{SH} > 0$. Hence, any solution (\hat{v}, \hat{w}) to (13) or (14) must be of the form $\hat{v} = v_\rho(\hat{w})$.

We claim that v_ρ is a non-decreasing function on $(0, \infty)$. Indeed, arguing by contradiction, suppose that $v_\rho(w') < v_\rho(w)$ for some $w' > w$. Then $v_\rho(w) \in [0, w]$ and there exists some $v \in (v_\rho(w'), v_\rho(w)] \neq \emptyset$ with $V_R(v, w) \leq 0$. By cash-invariance of ρ it follows $V_R(v, w') = V_R(v, w) + w - w' < V_R(v, w) \leq 0$, which again implies $v \leq v_\rho(w')$, a contradiction. Whence v_ρ is non-decreasing. Moreover, since ρ and thus V_R is convex, the function $v_\rho : (0, \infty) \rightarrow [-\infty, \infty)$ is concave, and thus continuous on the interior of its domain $\mathcal{D} = \{w \in (0, \infty) \mid v_\rho(w) > -\infty\}$ (see e.g. Rockafellar [19, Theorems 5.3 and 10.1]).

By assumption, \mathcal{D} is a non-empty interval with $\sup \mathcal{D} = \infty$. We now analyze the properties of the policy- and shareholder utility functions along the curve $(v_\rho(w), w)$ for $w \in \mathcal{D}$. First, we claim that $V_{PH}(v_\rho(w), w)$ is strictly quasiconcave and continuous in $w \in \mathcal{D}$. Indeed, let $w_1 < w_2 < w_3$ be points in \mathcal{D} . Since v_ρ is concave and non-decreasing, there exists some $\lambda \in (0, 1)$ such that $v_\rho(w_2) = \lambda v_\rho(w_1) + (1 - \lambda)v_\rho(w_3)$ and $w_2 \leq \lambda w_1 + (1 - \lambda)w_3$. From this, and since $\partial_w V_{PH} < 0$, we derive

$$V_{PH}(v_\rho(w_2), w_2) \geq V_{PH}(\lambda(v_\rho(w_1), w_1) + (1 - \lambda)(v_\rho(w_3), w_3)).$$

On the other hand, the policyholder utility function, $V_{PH}(a + bw, w)$, is strictly concave along straight lines of the form $(a + bw, w) \in \mathcal{V}$, for constant parameters a and b .⁹ Hence

$$V_{PH}(\lambda(v_\rho(w_1), w_1) + (1 - \lambda)(v_\rho(w_3), w_3)) > \min \{V_{PH}(v_\rho(w_1), w_1), V_{PH}(v_\rho(w_3), w_3)\}.$$

⁹The function $u(w_0 + c_0 - w - (X - w - (a + bw)R))^+$ is concave in w , and strictly concave in w for solvency states $X \leq w + (a + bw)R$. Taking expectation preserves the strict concavity since $\mathbb{P}[S(a + bw, w)] > 0$.

We thus obtain $V_{PH}(v_\rho(w_2), w_2) > \min \{V_{PH}(v_\rho(w_1), w_1), V_{PH}(v_\rho(w_3), w_3)\}$, which proves the quasiconcavity of $V_{PH}(v_\rho(w), w)$. The continuity follows from the continuity of v_ρ on \mathcal{D} . On the other hand, since $\partial_v V_{SH} > 0$, $\partial_w V_{SH} > 0$, and v_ρ is concave and non-decreasing, we conclude that $V_{SH}(v_\rho(w), w)$ is a strictly increasing continuous function in $w \in \mathcal{D}$.

As shown in Lemma A.3 (iii), the policyholder's level set $\{V_{PH} \geq \gamma_{PH}\}$ is compact in \mathcal{V} for any γ_{PH} . The theorem now follows by the afore proved properties of the policy- and shareholder utility functions along the curve $(v_\rho(w), w)$, $w \in \mathcal{D}$. \square

Remark 4.7. *We note that without convexity of ρ , the function $v_\rho(w)$ and thus $V_{PH}(v_\rho(w), w)$ and $V_{SH}(v_\rho(w), w)$ may fail to be continuous in w . Therefore the maximum of (13) or (14) may not be attained.*

Theorem 4.6 shows that there exists a unique solution to the risk shifting problem under the regulatory constraints. However, while the policyholder constraint is binding at this investment and premium policy, the shareholder's participation constraint may not. The regulatory constraint is binding for inner solutions ($0 \leq \hat{\alpha} < 1$), but not necessarily for $\hat{\alpha} = 1$. From the proof of Theorem 4.6 we obtain the following corollary.

Corollary 4.8. *Assume $\mathcal{D}_\alpha \neq \emptyset$. For any reservation utility level $\gamma_{SH} \in \Gamma_{SH}^{reg}$, there exists a unique policy $(\hat{\alpha}', \hat{p}')$ in \mathcal{P} which lies on the intersection of the shareholder level curve, $U_{SH}(\hat{\alpha}', \hat{p}') = \gamma_{SH}$, and the regulatory constraint set in (13). That is,*

$$U_{SH}(\hat{\alpha}', \hat{p}') = \max \{U_{SH}(\alpha', \hat{p}') \mid \rho(L(\alpha', \hat{p}')) \leq c_0\}.$$

The maximizer $(\hat{\alpha}, \hat{p})$ of (13) satisfies $U_{PH}(\hat{\alpha}, \hat{p}) \geq U_{PH}(\hat{\alpha}', \hat{p}')$.

We now consider the effect of solvency regulation on the agency cost by comparing the investment and premium policies and their implied welfare under the risk shifting problem without and with solvency regulation. While in view of Theorems 3.2 and 4.4, the shareholder's reservation utility constraint is binding without solvency regulation, it may not be binding under solvency regulation (see Theorem 4.6). However, as indicated in Corollary 4.8, it is sufficient for welfare comparison to compare the policyholder's utility as a function of $\alpha \in [0, 1]$ along the shareholder's respective level curve.

In the proof of Theorem 3.4, we have shown that this function can only assume three possible

shapes. It is either strictly increasing, or strictly decreasing, or attains a global maximum at a unique critical point $\alpha^* \in [0, 1]$ and is strictly increasing to the left and strictly decreasing to the right of α^* .

If the policyholder's utility along the shareholder's respective level curve is strictly increasing in α , then the investment policy under the risk shifting problem without solvency regulation, $\bar{\alpha} = 1$, is Pareto optimal, i.e. $\alpha^* = \bar{\alpha} = 1$. Solvency regulation by limiting the investment policy to $\hat{\alpha}'$ with

$$\hat{\alpha}' < \alpha^* = \bar{\alpha} = 1$$

reduces the policyholder's utility and thus welfare.

If the policyholder's utility along the shareholder's respective level curve attains a global maximum at a unique critical point $\alpha^* \in [0, 1)$, the risk shifting problem leads to a welfare loss. If solvency regulation implies an investment level $\hat{\alpha}'$ which is higher than the Pareto optimal level, i.e. if

$$\alpha^* \leq \hat{\alpha}' < \bar{\alpha} = 1,$$

then the policyholder's utility and thus welfare is higher under the regulatory constraint. This is because the policyholder's utility is strictly decreasing for all $\alpha \geq \alpha^*$. If solvency regulation is tighter and implies an investment level which is lower than the Pareto optimal level, i.e. if

$$\hat{\alpha}' < \alpha^* < \bar{\alpha} = 1,$$

then the impact of the regulatory constraint on welfare is ambiguous. This is because the policyholder's utility is strictly increasing for all $\alpha \leq \alpha^*$. In particular, for very tight solvency regulation that restricts the set of investment policies to very low levels, regulation can even further reduce welfare relative to the risk shifting problem without regulation.

In view of the dual problem in Theorem 4.6, the welfare effect of solvency regulation can be analogously discussed by comparing the shareholder's utility as a function of $\alpha \in [0, 1]$ along the policyholder's respective level curve. Since this function can also only assume the same three shapes (see Remark 3.5), we obtain the qualitatively identical results.

5 Numerical Example

In this section we first calibrate our model to the average portfolio of an European Economic Area non-life insurer taken from the Quantitative Impact Study 3 (QIS3) Benchmarking Study [4] of the Chief Risk Officer (CRO) Forum. From these data we derive all exogenous model parameters in our model. In particular, we determine the initial capital c_0 and the stochastic model for market risk R and insurance risk X . We then use our numerical findings to illustrate our analytical results.

The average stand alone capital requirements for stock market investment and insurance risk under the Solvency II standard model [2, 3] are

$$\text{SCR}_{mkt} = 2,508 \quad \text{and} \quad \text{SCR}_{ins} = 4,332,$$

respectively.¹⁰ Under Solvency II, the market investment and insurance risk are assumed to have a linear correlation coefficient of 0.25. Thus, the diversified total solvency capital requirement equals

$$\text{SCR}_{tot} = \sqrt{\text{SCR}_{mkt}^2 + 2 \times 0.25 \times \text{SCR}_{mkt} \text{SCR}_{ins} + \text{SCR}_{ins}^2} = 5,522.$$

The Solvency II risk measure ρ is the value-at-risk, $\text{VaR}_{99.5\%}$, at the 99.5% confidence level. Thus, the Solvency II stand alone capital requirement for market risk is determined by

$$\text{SCR}_{mkt} = \text{VaR}_{99.5\%}[\text{market loss} = -(c_0 + p_0) \alpha_0 R] \quad (15)$$

for some representative premium p_0 to be determined below, and the representative investment policy $\alpha_0 = 1/7$.¹¹ The Solvency II stand alone capital requirement for insurance risk equals

$$\text{SCR}_{ins} = \text{VaR}_{99.5\%}[\text{insurance loss} = X - p_0]. \quad (16)$$

The Solvency II test demands that the available capital, c_0 , be greater than or equal to the total

¹⁰These figures are derived from the proportion splits of QIS3 capital charges as shown on pages 39, 41, 43 in the document [4]. The capital requirements are thus normalized such that the undiversified total solvency capital requirement (SCR) results in 100×100 . The risk class “default” is negligible and the market risk types other than “equity” have been omitted for simplicity. The final numbers can be extracted from [6], Figure 2 where SCR_{ins} is derived from the SCR for premium risk and catastrophe risk which are assumed to be uncorrelated.

¹¹This number is derived from annual financial statements of non-life insurers.

solvency capital requirement, SCR_{tot} . We henceforth assume that

$$\text{SCR}_{tot} = c_0. \quad (17)$$

We now specify the stochastic model for market investment risk R and insurance risk X . We assume that

$$R = e^Y - 1 \quad \text{and} \quad X = e^Z,$$

where (Y, Z) is jointly normally distributed with mean (μ_Y, μ_Z) , standard deviations σ_Y, σ_Z , and a linear correlation of -0.25 .¹² Furthermore, we assume that

$$\mathbb{E}[R] = 0.04, \quad \text{var}[R] = (0.16)^2. \quad (18)$$

We use the following premium calculation principle to calibrate the insurance risk parameters

$$p_0 = \mathbb{E}[X]. \quad (19)$$

In solving Equations (15)–(19) we determine the parameters that fully specify our model $(c_0, \mu_Y, \mu_Z, \sigma_Y, \sigma_Z)$.¹³

As for the policyholder's utility function, we assume constant absolute risk aversion.¹⁴ Thus, the policyholder's expected utility is

$$U_{PH}(\alpha, p) = -e^{-\beta w_0} \mathbb{E} \left[e^{-\beta(-p - (X - (c_0 + p)(1 + \alpha R))^+)} \right],$$

where β denotes the coefficient of absolute risk aversion.

Our numerical results are presented in Figures 1, 2, and 3 in Appendix B for different degrees of risk aversion, $\beta = 10, 30$, and 130 , respectively. Computation shows that Assumption (4.1) (i) is satisfied for all three values of β . Moreover, Lemma A.5 ensures that also Assumption (4.1) (ii) holds. Thus, in view of Lemmas 4.2 and A.2, the shareholder's utility is increasing in α and p .

¹²This yields an approximate linear correlation of 0.25 between $-R$ and X .

¹³ For numerical reasons we normalize Equations (15)–(19) such that $c_0 + p_0 = 1$. This does not change the qualitative aspects of our numerical results.

¹⁴We choose constant absolute risk aversion since it allows for negative terminal wealth. Moreover, the risk preferences are independent of the policyholder's initial wealth w_0 . Below we choose w_0 in a way that it increases numerical precision.

The plots cover a section of the policy space \mathcal{P} where α is on the horizontal and p on the vertical axis. The thin solid and thin dashed lines depict the level curves of the shareholder (LC Sh) and policyholder (LC Ph) for utility levels γ_{SH} and γ_{PH} . The policyholder utility is maximal in the south-west corner of the plots. The thick line characterizes the policy strategies (α, p) that satisfy the first order condition (3) for Pareto optimality (FOC). We observe that in accordance with Theorems 3.3 and 3.4 there exists at most one Pareto optimum for every α that is unique for a specific reservation utility level.

The Pareto optimal policy under perfect competition is labeled with PC. It lies on the shareholder's reservation utility level that is equal to his outside option of not selling insurance, $\gamma_{SH} = c_0$. Analogously, MO labels the Pareto optimal policies in a monopolistic insurance market which is located on the policyholder's reservation utility level equal to his outside option of not buying insurance, that is, $\gamma_{PH} = \mathbb{E}[u(w_0 - X)] \in \Gamma_{PH}$ by Lemma A.3. In a frictionless market, depending on the level of competition the equilibrium solution lies on the FOC between PC and MO.

The dotted and slash-dotted lines are the boundaries of the regulatory constraints $\rho(L(\alpha, p)) \leq c_0$ under the value-at-risk measure $\text{VaR}_{99.5\%}$ (VaR) and the expected shortfall measure $\text{ES}_{99\%}$ (ES), respectively. The policies that are acceptable to the regulator are to the north-west of these boundaries. We note that the value-at-risk, $\text{VaR}_{99.5\%}$, does not satisfy the convexity property in Assumption 4.5 in general, see e.g. McNeil et al. [15]. However, in our example it shows convex behavior for the relevant values of (α, p) . For comparison, we also consider the Swiss Solvency Test [1] regulatory risk measure, which is the expected shortfall¹⁵, $\text{ES}_{99\%}$, at the 99% confidence level. The expected shortfall satisfies all properties of Assumption 4.5.

Figure 4 shows the regulatory constraints under the expected shortfall measure ES_q at different confidence levels $q = 99, 90$, and 60 . In a frictionless market with perfect competition the optimal investment and premium strategy (α^*, p^*) will be obtained at point PC. However, if shareholders cannot credibly commit to an investment strategy the optimal solution $(\bar{\alpha}, \bar{p})$ is attained at the risk shifted solution RS with $\bar{\alpha} = 1$ as implied by Theorem 4.4. This is harmful to the policyholder since the policyholder's utility decreases as (α, p) moves away from the PC along the shareholder's reservation utility curve. In this case regulation helps. Regulation restricts the set of feasible premium and investment strategies and might keep the shareholder from excessive risk taking. According to Theorem 4.6 there exists a unique solution $(\hat{\alpha}, \hat{p})$ for the regulated risk shifting problem. For

¹⁵Also known as conditional or tail value-at-risk.

different confidence levels these solutions are labeled R99, R90, and R60, respectively.¹⁶ As can be seen from Figure 4, regulation improves efficiency under the risk shifting problem for confidence levels $q \in [99, 90, 60]$. Among the regulatory measures presented in Figure 4, $ES_{90\%}$ is optimal. Solvency requirements are too tight for $ES_{99\%}$ and too weak for $ES_{60\%}$.

We have chosen constant absolute risk aversion because it eliminates wealth effects from our analysis. Unfortunately, the coefficient of absolute risk aversion is difficult to interpret since most empirical studies on estimating risk aversion are based on the assumption of constant relative risk aversion. Nevertheless, we can observe the behavior of the Pareto optimal point under perfect competition. We find that, with increasing degree of risk aversion β , the optimal investment in the stock market reduces while the optimal premium level increases. Moreover, in our example, the expected shortfall measure implies a more stringent regulatory requirement than the one implied by the value-at-risk measure. For lower degrees of risk aversion, $\beta = 10$ and $\beta = 30$, the Pareto optimal policies do not satisfy the regulatory constraints. For higher degrees of risk aversion, e.g. $\beta = 130$, PC meets the regulatory requirements.

6 Conclusion

In this paper, we provide a formal framework to analyze the conflict of interest policyholders and shareholders of insurance companies face. Increasing the risk of the insurer's assets and liabilities raises shareholder value potentially at the expense of policyholders. We characterize investment strategies and premium policies under Pareto optimality, under the risk shifting problem, and under solvency regulation. Moreover, we analyze the effect of solvency regulation on the agency cost of the risk shifting problem.

Solvency capital requirements limit the set of possible risk structures and thereby provide a commitment device for shareholders imposed by the regulator. There are other possible contractual or organizational arrangements that serve as commitment devices and thus reduce the agency cost. Examples include investment and dividend policy restrictions in corporate charters, issuing participating policies, or changing the organizational form to a mutual insurance company. While these contractual arrangements reduce the agency cost of the risk shifting problem they add other trade-offs. In this paper, we took the insurance contract as given and explored how solvency

¹⁶The numerical values for α , p and U_{PH} at points of interest can be looked up in Table 1.

regulation might address the risk shifting problem without distorting insurance contracts.

A Appendix: Lemmas

Lemma A.1. (i) $U_{SH}(\alpha, p)$ is convex in α and in p .

(ii) $U_{PH}(\alpha, p)$ is concave in α and strictly concave in p .

Proof. For fixed $R = r$ and $X = x$, the function $((c_0 + p)(1 + \alpha R) - X)^+$ is convex in p and in α . Moreover, $u(w_0 - p - (X - (c_0 + p)(1 + \alpha R))^+)$ is concave in p and in α , and strictly concave in p for solvency states $X \leq (c_0 + p)(1 + \alpha R)$. Taking expectation preserves these properties since $\mathbb{P}[\mathcal{S}(\alpha, p)] > 0$. \square

Lemma A.2. The derivatives of U_{SH} and U_{PH} are given by:

$$\begin{aligned}\partial_\alpha U_{SH}(\alpha, p) &= (c_0 + p) \mathbb{E} [R 1_{\mathcal{S}(\alpha, p)}] \\ \partial_p U_{SH}(\alpha, p) &= \mathbb{E} [(1 + \alpha R) 1_{\mathcal{S}(\alpha, p)}] > 0 \\ \partial_\alpha U_{PH}(\alpha, p) &= (c_0 + p) \mathbb{E} [R u'(w_0 - p - X + (c_0 + p)(1 + \alpha R)) 1_{\mathcal{S}(\alpha, p)^c}] \\ \partial_p U_{PH}(\alpha, p) &= -u'(w_0 - p) \mathbb{P}[\mathcal{S}(\alpha, p)] \\ &\quad + \alpha \mathbb{E} [R u'(w_0 - p - X + (c_0 + p)(1 + \alpha R)) 1_{\mathcal{S}(\alpha, p)^c}]\end{aligned}$$

for all $(\alpha, p) \in \mathcal{P}$.

Proof. We can write

$$\begin{aligned}U_{SH}(\alpha, p) &= \int_{-\infty}^{\infty} \int_{-\infty}^{(c_0 + p)(1 + \alpha r)} ((c_0 + p)(1 + \alpha r) - x) f(x, r) dx dr \\ U_{PH}(\alpha, p) &= \int_{-\infty}^{\infty} \int_{-\infty}^{(c_0 + p)(1 + \alpha r)} u(w_0 - p) f(x, r) dx dr \\ &\quad + \int_{-\infty}^{\infty} \int_{(c_0 + p)(1 + \alpha r)}^{\infty} u(w_0 - p - x + (c_0 + p)(1 + \alpha r)) f(x, r) dx dr\end{aligned}$$

Hence the assertion follows by straightforward formal differentiation as justified by Assumption 2.1.

That $\partial_p U_{SH}(\alpha, p) > 0$ follows from Assumption 2.1(ii). \square

Lemma A.3. (i) $U_{SH}(\alpha, -c_0) \equiv 0$ and $\lim_{p \rightarrow \infty} \inf_{\alpha \in [0, 1]} U_{SH}(\alpha, p) = \infty$. Hence the range is

$$U_{SH}(\mathcal{P}) = (0, \infty).$$

(ii) For every level $\gamma_{SH} \in (0, \infty)$, the level curve $\{U_{SH} = \gamma_{SH}\}$ is a C^1 -line in \mathcal{P} connecting the $\{\alpha = 0\}$ - and $\{\alpha = 1\}$ -axes.

(iii) $U_{PH}(\alpha, -c_0) \equiv \mathbb{E}[u(w_0 + c_0 - X)]$ and $\lim_{p \rightarrow \infty} \sup_{\alpha \in [0,1]} U_{PH}(\alpha, p) = -\infty$.

(iv) U_{PH} attains its global supremum at $(1, p_{PH}^{crit})$. That is,

$$U_{PH}(1, p_{PH}^{crit}) = \sup_{(\alpha, p) \in \mathcal{P}} U_{PH}(\alpha, p). \quad (20)$$

Moreover, $p_{PH}^{crit} > -c_0$ if and only if $\mathbb{E}[R u'(w_0 + c_0 - X)] > 0$.¹⁷ In this case, $(1, p_{PH}^{crit})$ is the unique maximizer for U_{PH} in \mathcal{P} . Hence the range is $U_{PH}(\mathcal{P}) = \Gamma_{PH}$.

(v) For every level $\gamma_{PH} \in \Gamma_{PH}$, the level curve $\{U_{PH} = \gamma_{PH}\}$ is a C^1 -line in \mathcal{P} , connecting the $\{\alpha = 0\}$ - and $\{\alpha = 1\}$ -axes if $\gamma_{PH} \leq \mathbb{E}[u(w_0 + c_0 - X)]$.

Proof. Most of the stated properties follow straightforward from the definition of U_{SH} and U_{PH} , and Lemmas A.1 and A.2. We only give details for some of the statements.

Jensen's inequality implies $U_{SH}(\alpha, p) \geq (c_0 + p)(1 + \alpha \mathbb{E}[R]) \geq (c_0 + p) \min\{1, 1 + \mathbb{E}[R]\}$. Since $\mathbb{E}[R] > -1$ we conclude that $\lim_{p \rightarrow \infty} \inf_{\alpha \in [0,1]} U_{SH}(\alpha, p) = \infty$, which proves (i).

Next, note that $U_{PH}(\alpha, p) \leq \mathbb{E}[u(w_0 - p)]$, hence $\lim_{p \rightarrow \infty} \sup_{\alpha \in [0,1]} U_{PH}(\alpha, p) = -\infty$, and U_{PH} is uniformly bounded on \mathcal{P} in particular, whence (iii).

In view of Lemma A.2, U_{PH} cannot have a critical point in the interior of \mathcal{P} , and $U_{PH}(0, p)$ is strictly decreasing in p . On the other hand, $U_{PH}(\alpha, -c_0) \equiv \mathbb{E}[u(w_0 + c_0 - X)]$ is constant in $\alpha \in [0, 1]$. We conclude that $U_{PH}(\alpha, p)$ attains its maximum in $(1, p_{PH}^{crit})$, which proves (20). Next we note that $\mathbb{P}[\mathcal{S}(\alpha, -c_0)] = \mathbb{P}[X \leq 0] = 0$. Hence Lemma A.2 implies that $\partial_p U_{PH}(\alpha, -c_0) = \alpha \mathbb{E}[R u'(w_0 + c_0 - X)]$, and thus $p_{PH}^{crit} > -c_0$ if and only if $\mathbb{E}[R u'(w_0 + c_0 - X)] > 0$, whence (iv).

The smoothness properties (ii) and (v) of the level curves follows by the implicit function theorem and the fact that ∇U_{SH} and ∇U_{PH} are nowhere zero in \mathcal{P} . Finally, $U_{PH}(0, p)$ and $U_{PH}(1, p)$ are strictly decreasing in $p \in (-c_0, \infty)$ and $p \in (p_{PH}^{crit}, \infty)$, respectively, with $U_{PH}(0, -c_0) = \mathbb{E}[u(w_0 + c_0 - X)] \leq \gamma_{PH}^{crit} = U_{PH}(1, p_{PH}^{crit})$. Whence level curve $\{U_{PH} = \gamma_{PH}\}$ connects the $\{\alpha = 0\}$ - and $\{\alpha = 1\}$ -axes if $\gamma_{PH} \leq \mathbb{E}[u(w_0 + c_0 - X)]$. The statement on shareholder level curves follows similarly. \square

¹⁷A sufficient condition for this to hold is $\mathbb{E}[R | X = x] > 0$ for all $\text{ess inf } X < x < \text{ess sup } X$.

Lemma A.4. *The derivatives of V_{SH} and V_{PH} defined in (4) are given by:*

$$\begin{aligned}\partial_v V_{SH}(v, w) &= \mathbb{E} [R 1_{\mathcal{S}(v, w)}] \\ \partial_w V_{SH}(v, w) &= \mathbb{P} [\mathcal{S}(v, w)] > 0 \\ \partial_v V_{PH}(v, w) &= \mathbb{E} [R u'(w_0 + c_0 - X + vR) 1_{\mathcal{S}(v, w)^c}] \\ \partial_w V_{PH}(v, w) &= -u'(w_0 + c_0 - w) \mathbb{P} [\mathcal{S}(v, w)] < 0\end{aligned}$$

for all $0 \leq v \leq w$, $w \in [c_0, \infty)$.

Proof. Follows from Lemma A.2 and Assumption 2.1. □

Lemma A.5. *Let*

$$R = e^Y - 1 \quad \text{and} \quad X = e^Z,$$

where (Y, Z) is jointly normal distributed with mean (μ_Y, μ_Z) , standard deviations σ_Y, σ_Z , and linear correlation coefficient $\rho_{(Y, Z)}$. If $\mathbb{E}[R] > 0$ and $\rho_{(Y, Z)} < 0$, then

$$\mathbb{E}[R \mid X \leq c] > 0 \quad \text{for all } c \in (0, \infty).$$

Proof. The conditional distribution of Y given Z is normally distributed with mean $\mu_{Y|Z=z} = \mu_Y + \rho_{(Y, Z)} \frac{\sigma_Y}{\sigma_Z} (z - \mu_Z)$ and variance $\sigma_{Y|Z=z}^2 = \sigma_Y^2 (1 - \rho_{(Y, Z)}^2)$, see e.g. McNeil et al. [15, p. 68]. Hence

$$\mathbb{E}[R \mid X = x] = \int_{-\infty}^{\infty} (e^y - 1) \phi_{\mu_{Y|Z=\ln(x)}, \sigma_{Y|Z=\ln(x)}^2}(y) dy$$

where $\phi_{\mu, \sigma}$ denotes the normal density function with mean μ and variance σ^2 . Since $(e^y - 1)$ is strictly increasing in y , $\partial_x \mu_{Y|Z=\ln(x)} \leq 0$, and $\partial_x \sigma_{Y|Z=\ln(x)}^2 = 0$, we infer that $\partial_x \mathbb{E}[R \mid X = x] \leq 0$. Suppose, by contradiction, there exists some $c \in (0, \infty)$ with $\mathbb{E}[R \mid X \leq c] \leq 0$. Then $\partial_x \mathbb{E}[R \mid X = x] \leq 0$ implies that $\mathbb{E}[R \mid X = c] \leq 0$ and thus $\mathbb{E}[R \mid X = x] \leq 0$ for all $x \geq c$. Hence $\mathbb{E}[R] = \mathbb{E}[R \mid X \leq c] \mathbb{P}[X \leq c] + \mathbb{E}[R \mid X > c] \mathbb{P}[X > c] \leq 0$ contradicts $\mathbb{E}[R] > 0$. □

B Appendix: Figures and Table

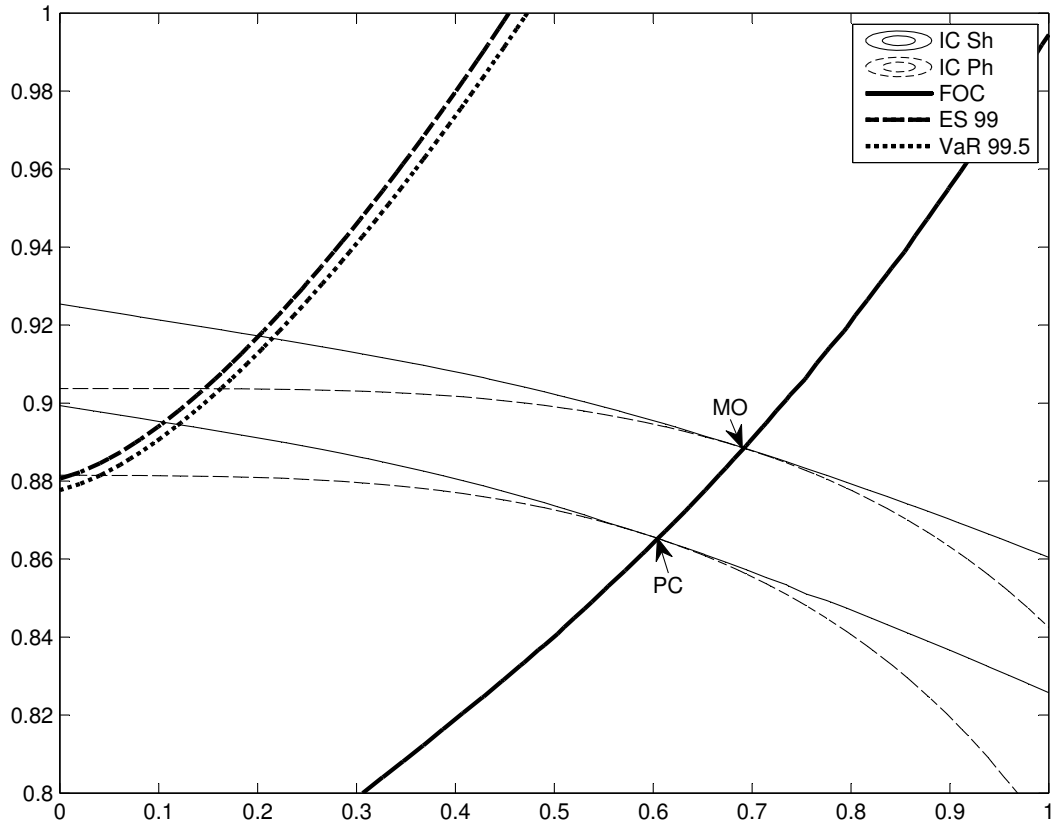


Figure 1: This Figure shows the numerical results for a coefficient of absolute risk aversion $\beta = 10$ in the (α, p) -space. The thin solid lines represent the shareholder's level curve (LC Sh). The thin dashed lines represent the policyholder's level curve (LC Ph). The contract curve (FOC) is depicted by the thick solid line. The points PC and MO denote the Pareto optimal policies in a perfectly competitive and in a monopolistic market, respectively. The thick dotted and slash dotted lines determine the set of feasible policies under the $ES_{99\%}$ -measure (ES) and the $VaR_{99.5\%}$ -measure (VaR).

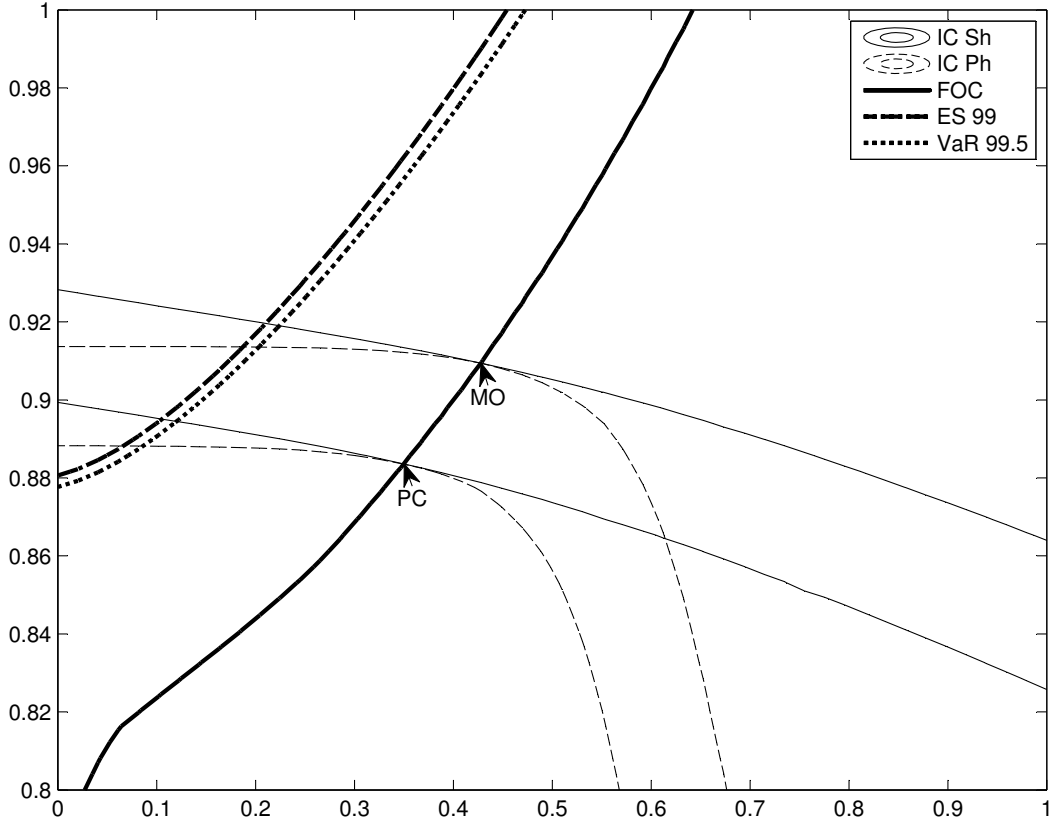


Figure 2: This Figure shows the numerical results for a coefficient of absolute risk aversion $\beta = 30$ in the (α, p) -space. The thin solid lines represent the shareholder's level curve (LC Sh). The thin dashed lines represent the policyholder's level curve (LC Ph). The contract curve (FOC) is depicted by the thick solid line. The points PC and MO denote the Pareto optimal policies in a perfectly competitive and in a monopolistic market, respectively. The thick dotted and slash dotted lines determine the set of feasible policies under the $ES_{99\%}$ -measure (ES) and the $VaR_{99.5\%}$ -measure (VaR).

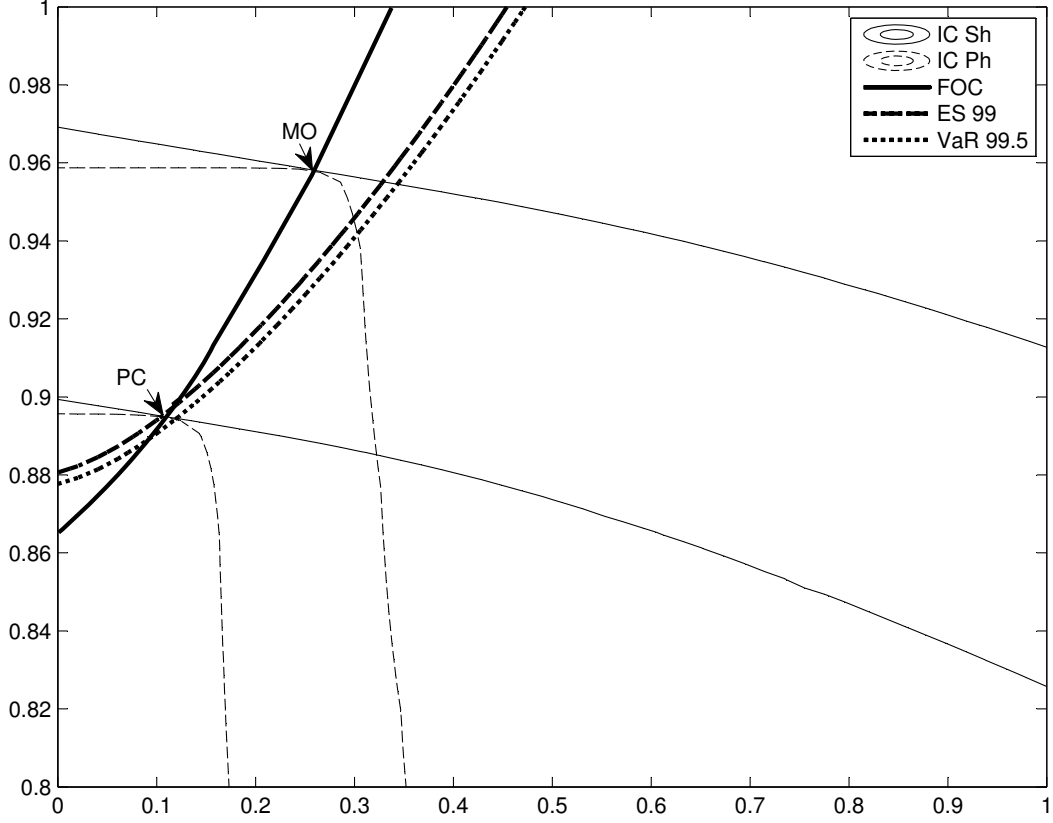


Figure 3: This Figure shows the numerical results for a coefficient of absolute risk aversion $\beta = 130$ in the (α, p) -space. The thin solid lines represent the shareholder's level curve (LC Sh). The thin dashed lines represent the policyholder's level curve (LC Ph). The contract curve (FOC) is depicted by the thick solid line. The points PC and MO denote the Pareto optimal policies in a perfectly competitive and in a monopolistic market, respectively. The thick dotted and slash dotted lines determine the set of feasible policies under the $ES_{99\%}$ -measure (ES) and the $VaR_{99.5\%}$ -measure (VaR).

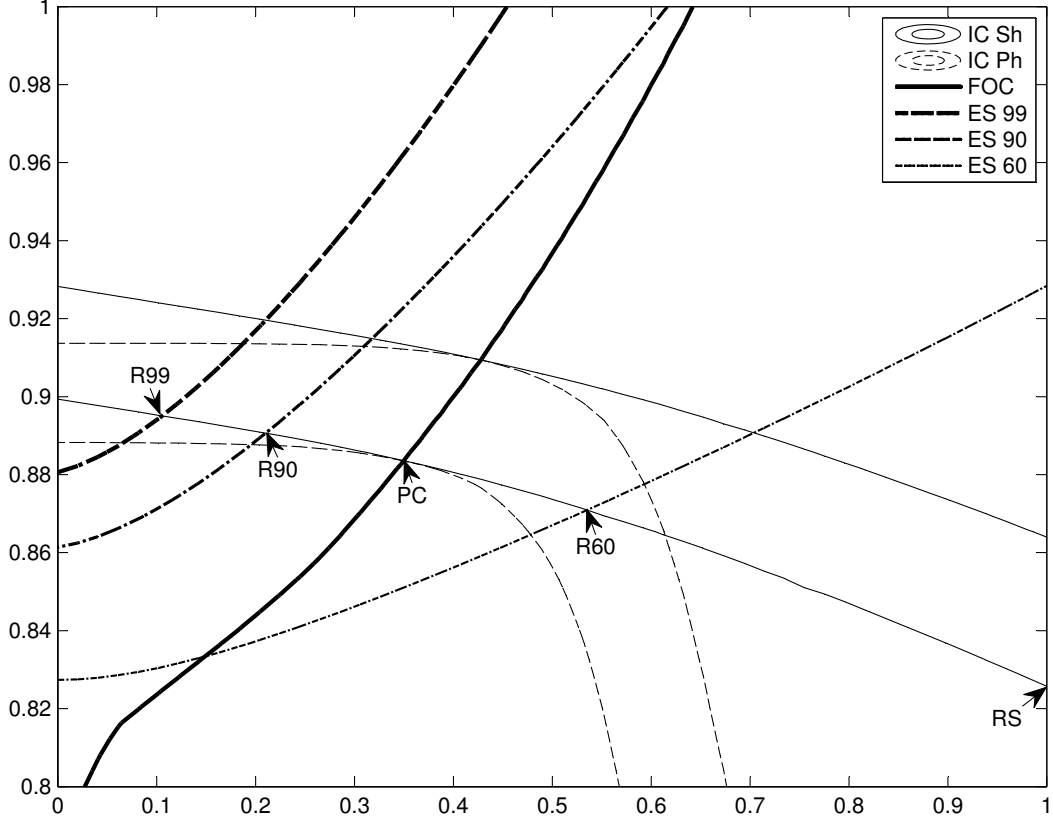


Figure 4: This Figure shows the regulatory constraints for a coefficient of absolute risk aversion $\beta = 30$ under the expected shortfall measure ES_q at different confidence levels $q = 99, 90$, and 60 for the case of perfect competition. That is, on the shareholder level curve $\gamma_{SH} = c_0$. In this setting the Pareto optimal policy (α^*, p^*) is labeled with PC, the risk shifted solution is denoted by RS, and the solutions to the regulated risk shifting problem for different confidence levels are marked with R99, R90, and R60, respectively.

	α	p	$U_{PH}(\alpha, p)$
PC = (α^*, p^*)	0.347	0.883	-1.00E-06
R99 = $(\hat{\alpha}_{ES_{99\%}}, \hat{p}_{ES_{99\%}})$	0.102	0.893	-1.14E-06
R90 = $(\hat{\alpha}_{ES_{90\%}}, \hat{p}_{ES_{90\%}})$	0.204	0.890	-1.02E-06
R60 = $(\hat{\alpha}_{ES_{60\%}}, \hat{p}_{ES_{60\%}})$	0.531	0.869	-1.31E-06
RS = $(\bar{\alpha}, \bar{p})$	1	0.824	-4.21E-05

Table 1: This Table shows the values of the policy (α, p) and the implied utility of the policyholder for points of interest marked in Figure 4.

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